



Math for Olympiad: A didactic proposal from the perspective of the International Mathematical Olympiad with GeoGebra software

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Abstract

The concept of the International Mathematics Olympiad (IMO) in learning mathematics in the classroom is something that can inspire students by structuring didactic proposals. This study aims to present a didactic proposal from the perspective of the International Mathematics Olympiad, using digital technology, particularly the GeoGebra software to be included in the mathematics Olympiad discipline. For theoretical studies, the analysis of the mathematical elements of the structured images in the GeoGebra software and the identification of their properties is included, through the structuring of the Olympic teaching situation, which allows the subject to seek solutions to the posed mathematical problems. olympics and validating the teaching of geometry. This study describes the arrangements made to build Olympic math objects that will be applied by math teachers. The research methodology is based on bibliographic reviews from authors such as Alves, Santiago, Almouloud and Brousseau. It is proven that the GeoGebra software helps in the elaboration of mathematical examples and problem solving situations.

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INTRODUCTION

The International Mathematics Olympiad (IMO) was formed in 1959 (Moroianu, 2021). Since then, there has been an increase in the participation of several countries in international Olympiad tournaments, followed by mathematics training for students selected by the delegations of each country to take part in the competition (Petersen & Wulff, 2017). The aim of implementing IMO is to discover, inspire and challenge students to understand mathematics better in an international setting. In that way, bonds of international friendship were ensured between mathematicians from all the participating countries, and created opportunities for the exchange and dissemination of mathematics in general (Choi, 2009; Jung & Lee, 2021). This is a great opportunity for students to develop mathematics widely.

The questions in the implementation of IMO must cover a variety of mathematical topics, bearing in mind that the questions presented in the olympiad test require good interpretation of olympiad questions and mathematical skills in applying the formulas that will be applied in solving the problems (Bankov, 2022). In recent years, problems have been classified in several areas of teaching mathematics, such as: geometry, number theory, algebra, and combinatorics (Andrews, 1986; Ono, 2008). The process of learning mathematics that refers to IMO goals is considered good to be applied in classroom learning, activities in the form of knowledge construction for use in a classroom education environment can encourage students to make their own discoveries and develop strategies by exploring the construction of mathematical solutions. The relevance of the issues raised in IMO and their pedagogical treatment in teaching mathematics is well known by the academic/scientific community (He et al., 2022; Saul & Vaderlind, 2022), but in practice schools need

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to be included in pedagogical planning to increase knowledge in solving problems with Olympiad issues (Lucena et al., 2022).

Math for the Olympics is part of its math and technology formative learning plan. Formative lesson plans are structured to stimulate and promote math olympiad learning in the classroom and prepare the learning stages for collaborating on solutions to everyday problem situations (Da Luz Moreira & Navarini, 2022; Iqbal et al., 2021). Objects of mathematical knowledge in IMO are presented in Table 1.

Table 1. Mathematical knowledge objects for Olympiads.

Arithmetic	Algebra	Counting	Number Theory
Geometry	FinancialMath	Techniques for Problem Solving	Digital Technologies

The object of mathematical knowledge for the Olympiad will be a reference for students to study material within the scope of IMO. All students have a greater or less tendency to participate in math competitions, depending on the ability of students to solve problems. Thus, the ability of students to solve problems becomes an ability that must be cultivated by students. Preparation in planning learning at school is first seen based on the facilities and tools that stimulate students' mathematical abilities (Garland & Tadeja, 2013). Because of this, an Olympic Didactic Situation (ODS) is needed which develops from an Olympic teaching situation, is validated and proven based on the TDS (Theory of Didactic Situations) dialectical phase conducted by Alves (2021) and will be used to support Olympiad activities taught by mathematics teachers in class with an emphasis on visualization components, supported by the software. International Olympiad situation planning, under the assumption of TSD, will give teachers control over the obstacles that students may face with doubts and questions, using means, interaction, and encouragement to students, expanding their knowledge of problem solving and qualifying. Mediation with the subject matter, especially in the context of the international mathematics olympiads, as the questions require deeper understanding and allow different ways of achieving results through writing and digital technology support (Steeh et al., 2021).

The arrangement of mathematics proposals for Olympiads in teaching in the classroom is based on TSD (Choi et al., 2019), didactic situations are conceptualized by the emergence of pedagogic aspects between teachers, students, and mathematical knowledge (knowing) that are interconnected with teaching and learning situations, taking into account the discussion environment (Da Silva Santiago & Alves, 2022). Almouloud (2010), explained that the didactic environment and situation must include the mathematical knowledge involved in the student teaching and learning process. This learning situation brings several problems, which may be analyzed by TDS, through the analysis and visualization of interpretations defined by students, teachers, and knowledge. Thus, TDS is important in how to interpret the work constructed by students (Artigue et al., 2014; West et al., 2020), thereby stimulating the whole class to become participatory and interactive for the development of new mathematical knowledge.

According to (Alves, 2019) research assumptions in French studies, mathematics with didactic situations is a strong trend in the production of academic studies and the learning system investigation. The application of didactic situations by utilizing technology can also help students to be able maintain perspectives in mathematical characteristics (Novita & Herman, 2021; Rasmussen, 2016; Santiago & Alves, 2021). Other research findings conducted by Solares-Rojas et al., (2022) showing the use of didactic proposals can help educators in designing mathematical learning construction activities, and can develop students' thinking skills in problem solving (Fonseca & Arezes, 2017). Based on several studies research that has been done conducted and showing positive effect, it has not been found research related to didactic proposals that refer to the IMO perspective supported by GeoGebra software. This research can later be used as teaching technology to help them visualize numbers, streamline and predict potential new strategies for teaching mathematics and can use olympiad questions to teach plane geometry in digital-enabled classrooms. Therefore, the aim of this article is to present a didactic proposal for international olympiads based on the perspective of the International Mathematical Olympiad, using digital technology, more specifically the GeoGebra software for inclusion in olympic disciplines.

METHOD

The research methodology is based on bibliographic reviews from authors such as Alves, Santiago, Almouloud and Brousseau. The material that has been discussed regarding the didactic situation of the Olympics is divided into national and international, with the aim of finding the necessary subsidies for the development of activity proposals in class learning and preparation for the Mathematics Olympiad. Thus, a bibliographical review of the articles with the theme "The Didactic Situation of the International Olympiad" was carried out. The situation theory used goes through four stages, namely action, formulation, validation, and institutionalization.

Theoretical studies, analysis of mathematical elements of structured images in GeoGebra software and identification of their properties are included, through structuring olympic teaching situations, which enable subjects to find solutions to mathematical problems posed for olympics and validate teaching of geometry.

RESULTS AND DISCUSSION

It is seen that technological advances exist in teaching and play an important role in the transition of the teacher's role (Artigue et al., 2014). The following describes the necessary procedures and possible methods that can be carried out using the geometric constructions visualized in the GeoGebra software. According to Artigue (1988), didactic situations must be designed in such a way that student behavior can be predicted.

In the action phase, students can analyze understanding of international olympics, build their own decisions and at the end of each mathematical analysis, they observe the answers developed. Interaction must occur centrally in decision making between students and their peer groups, this class knowledge is part of each student's environment. This situation is the way in which students will organize their studies from Olympic math problem solving strategies.

An arbitrary point M is selected on the inside of segment AB : The $AMCD$ and $MBEF$ boxes are built on the same side of AB ; with AM and MB segments as the basis respectively. The circle that surrounds this square, with centers P and Q ; intersect at M as well as at another point N : Let N' be the point of intersection of the lines AF and BC :

(a) Prove that points N and N' coincide. In the first construction of the ODS, we observe two segments constructed according to Figure 1.

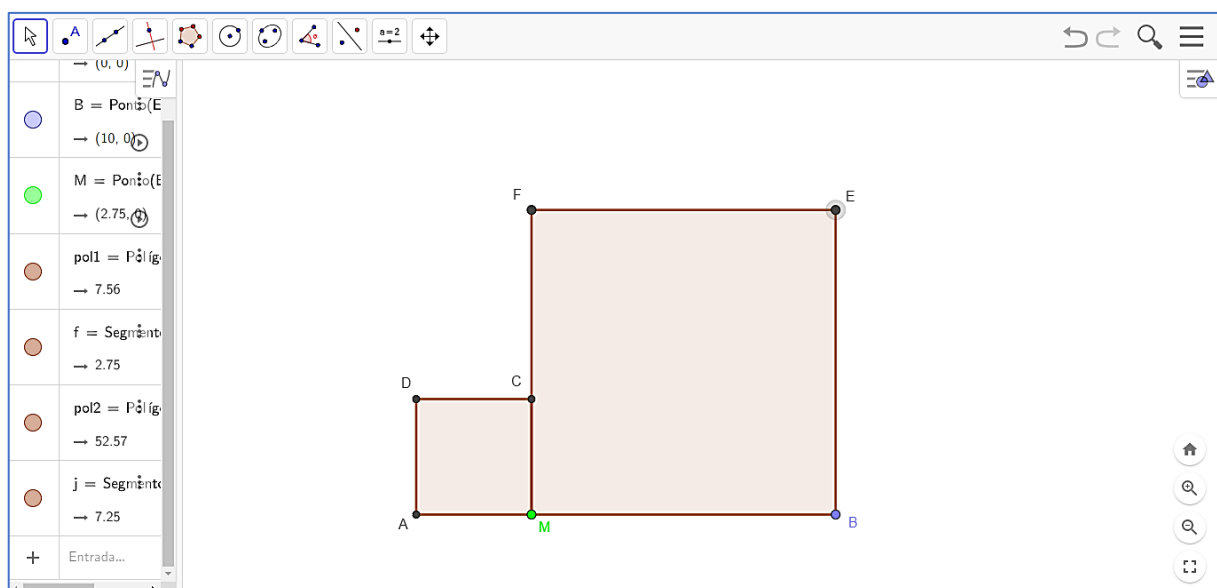


Figure 1. Segments connected to the ADCM and MBEF quadrilaterals

The formulation phase, the subject's ability to recover knowledge, exchange information through formal mathematical language that everyone can understand. Students exchange

information with other participants, they will only be senders and recipients of information on the Olympic didactic situation, exchanging some writings or speeches each occurring naturally, or mathematically.

Tracing the center points of the quad, we have a point FB that intersects Q and a point DM that passes through P:

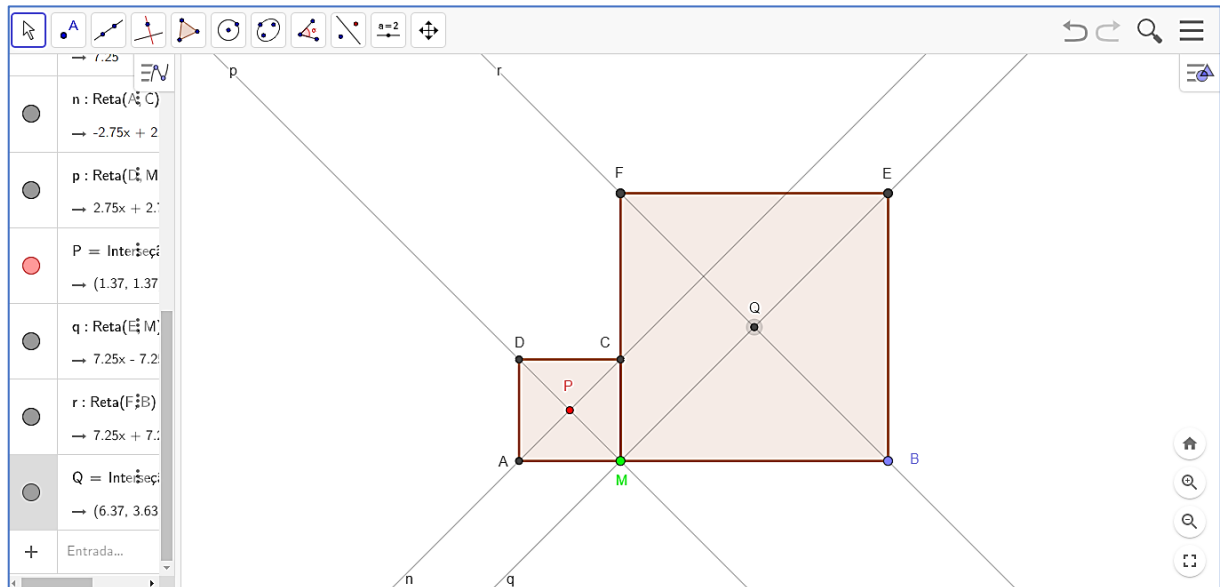


Figure 2. Incenter P and Q

The third validation step, it is possible to distinguish a new model of mathematical representation: the sender is no longer the announcer of the action, but the supporter, and the receiver is the opponent. The two work together to find the right solution, fulfilling existing knowledge areas.

According to Suleiman (2015) highlights that in TSD, students become protagonists of the characteristics of their didactic situations, or the relationship between students and the school environment is divided into three categories: a) exchange of undiscovered knowledge or language (actions and solutions); b) exchange of knowledge (information) encoded in mathematical language (explain); c) exchange of information about resolutions. In the construction of Figure 3, two circles bounded by a rectangle can be visualized, with their meeting point at point M.

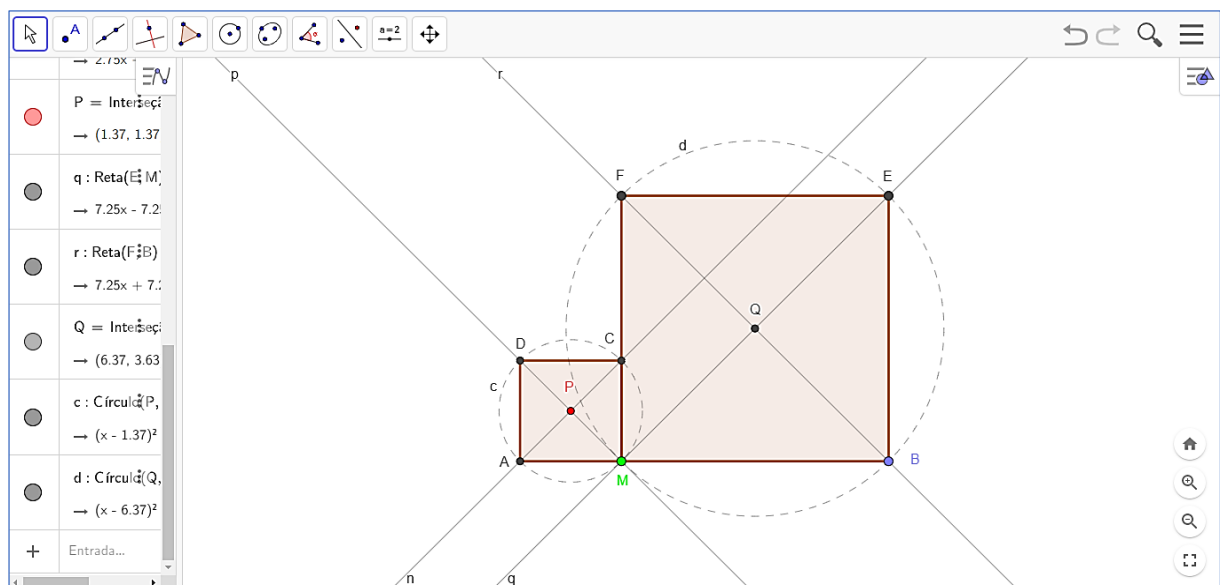


Figure 3. The bounded circle

In this way, students will arrive at the final solution presented in the table and will provide possible justifications for summarizing the reasons. GeoGebra software will play an important role in this matter to arrive at this result.

The last phase of institutionalization, referring to the steps shown in Figure 3, through the visualization of the GeoGebra software window, places the teacher in a commitment to action, identifying knowledge for the final consideration of all the previous steps, important and unique to acquire knowledge of subject conditioning.

Note that Figure 4, the parameterization of two straight lines s and t , has a point that intersects at N , passing through another point F , A , C and B .

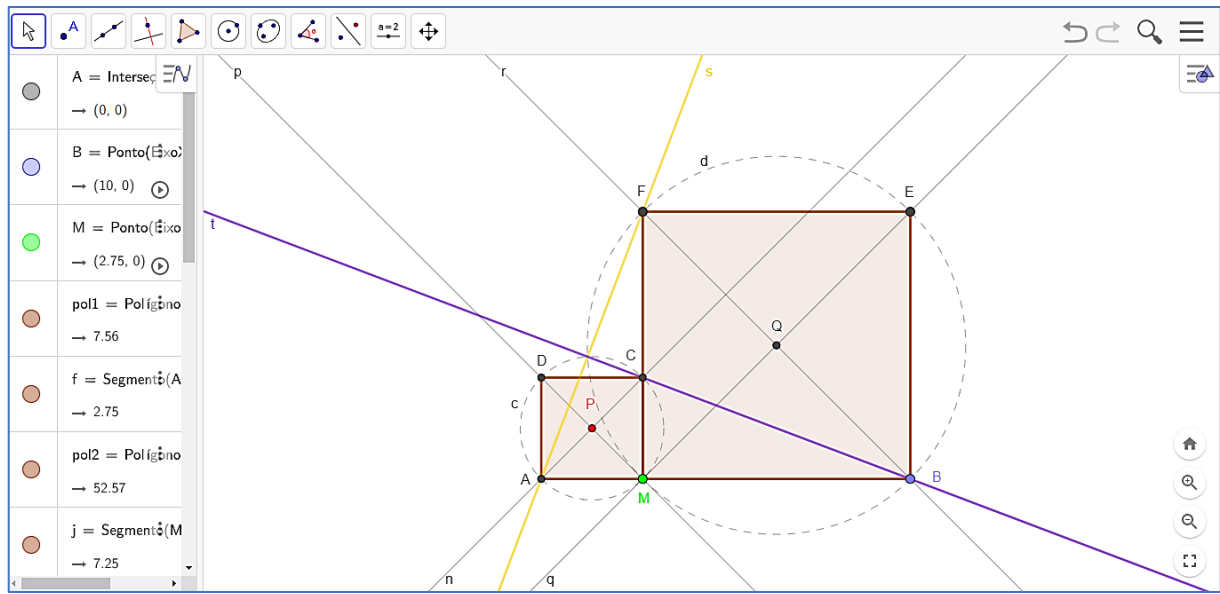


Figure 4. Congruent triangles

Since AFM is triangular, CBM is congruent, angles AFM, CBM are congruent; therefore, $\angle A'NB$ is a right angle. Therefore, N' must lie on the outer circle of the two quadrilaterals; therefore, it is the same point as N .

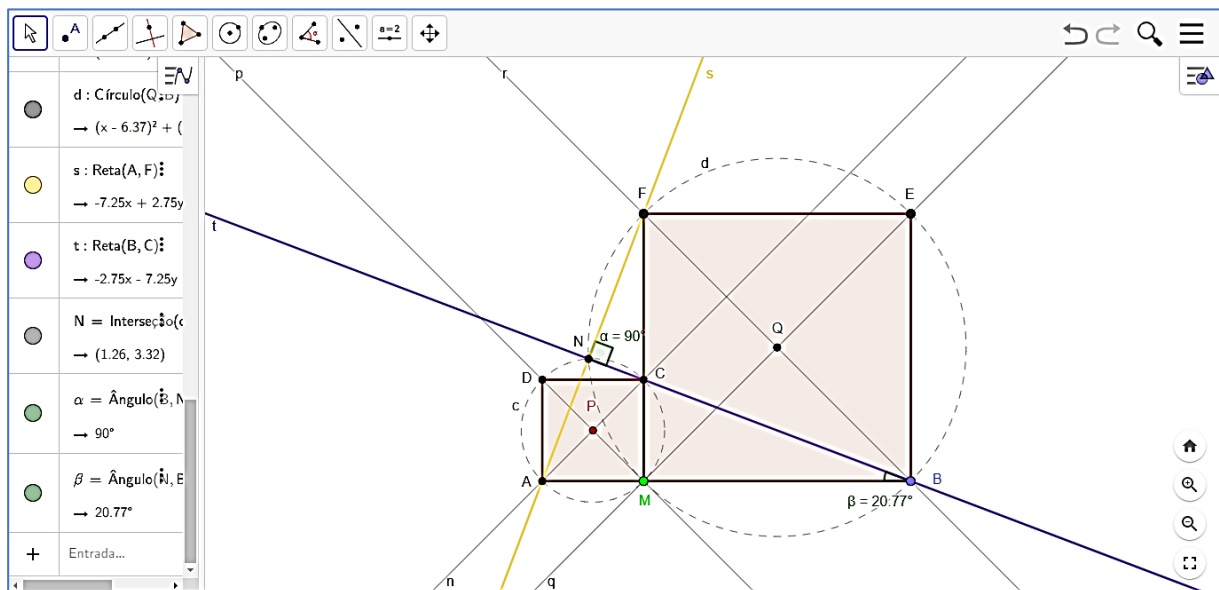


Figure 5. Inscribed quadrangle

The GeoGebra construction protocol is a dynamic table detailing all the commands performed in constructing an IMO Olympiad teaching situation.

Figure 6. Construction Protocol section 1

(b) Prove that the lines MN pass through fixed points S independent of M's choices:

We observe that $\frac{AM}{MB} = \frac{CM}{MB} = \frac{AN}{NB}$, looking at triangles ABN, BCM are similar. So, NM goes to ANB bisector.

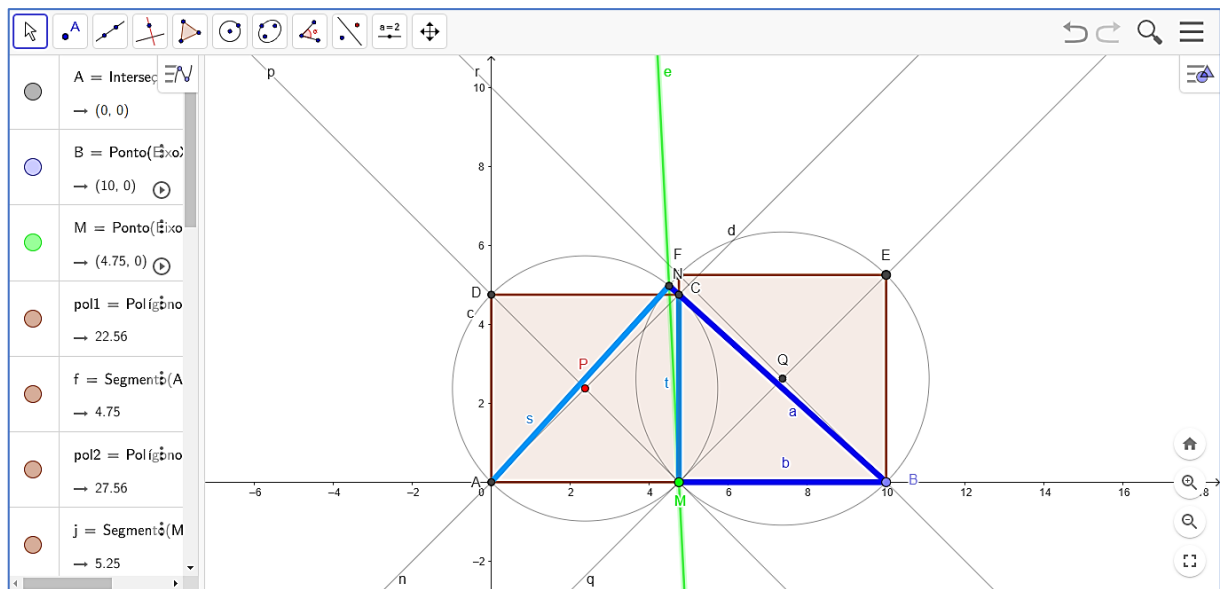


Figure 7. SDO construction

The parameterized line coincides with the segments starting from A, N, B and M. The points are connected to form a right angle.

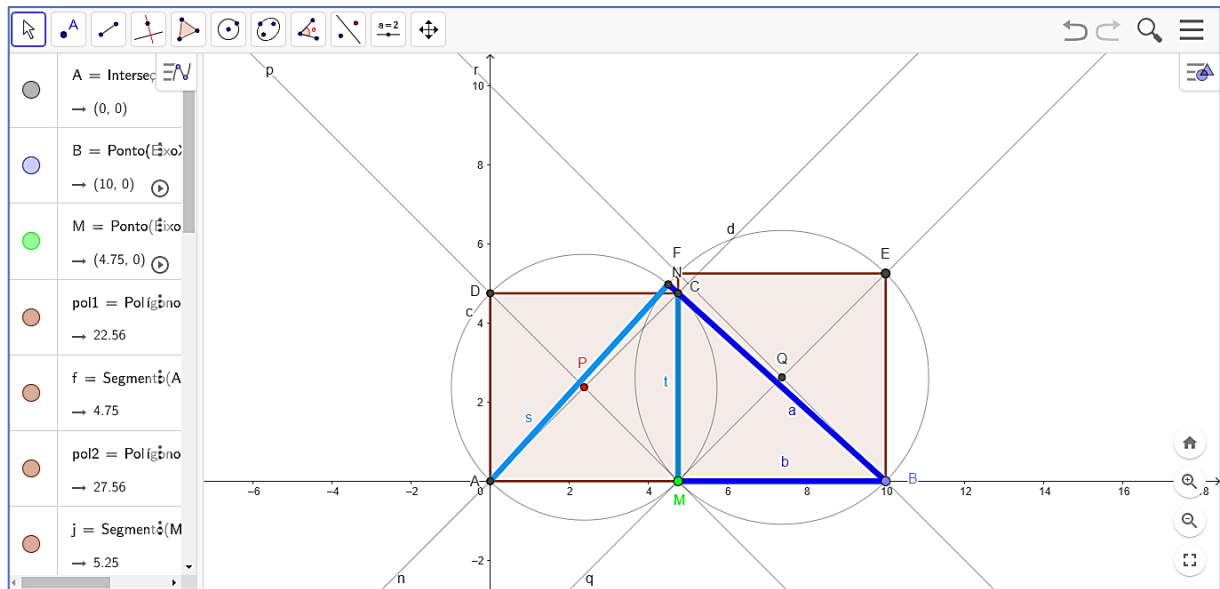


Figure 8. Intersecting points at AFB

To create another right angle, you create two segments in the AMCD quadrilateral, as illustrated in Figure 9.

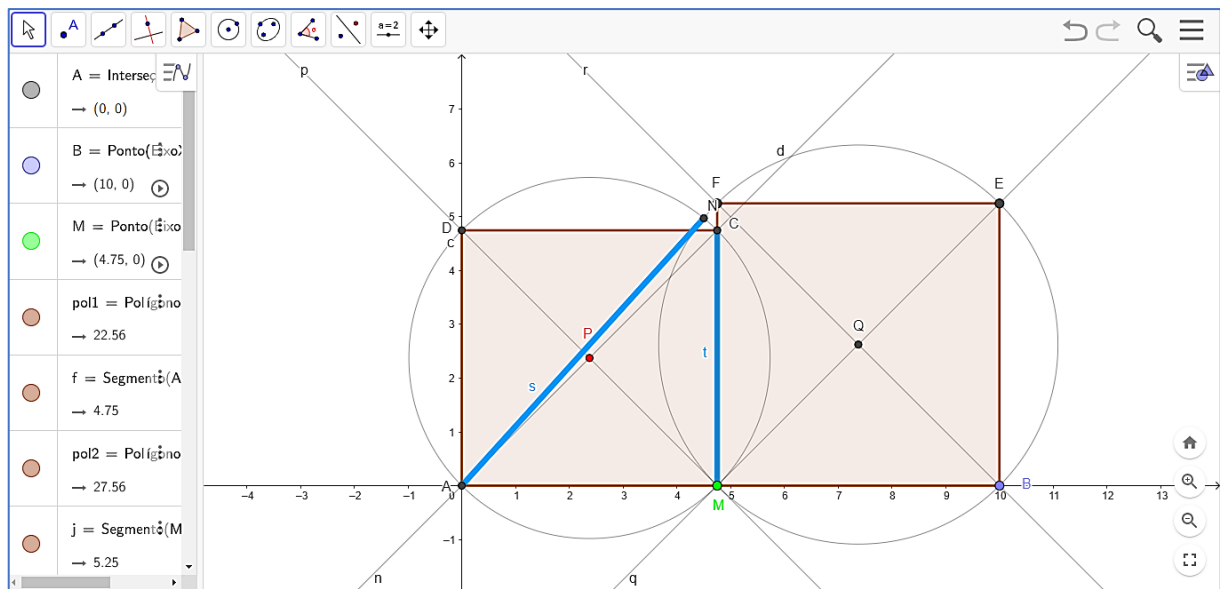


Figure 9. The quadrilateral bounded by AN and MC segments

Consider a circle with diameter AB. Since ANB is a right angle, N is on the circle, and because MN intersects ANB, the arcs it intersects are congruent, it passes through the bisector of arc AB (counterclockwise), becoming a constant point.

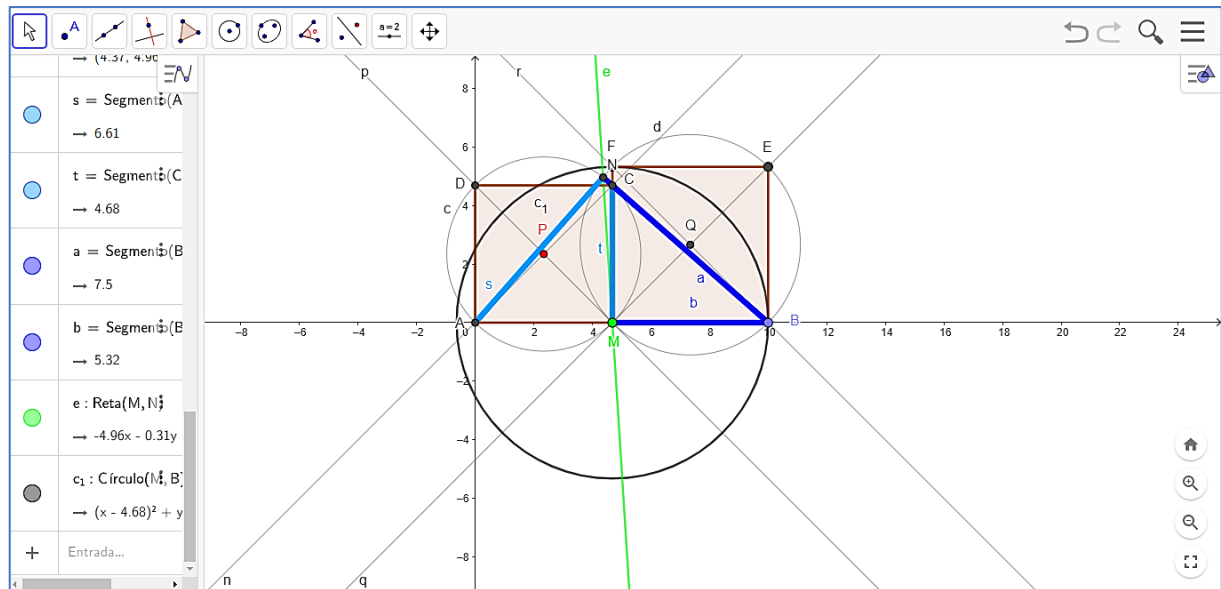


Figure 10. Perimeter parallel quadrilateral

With the didactic proposal, the construction protocol in the GeoGebra software is presented to be applied in Olympic mathematics classes.

Nº	Nome	Descrição	Valor
1	Ponto A	Interseção de EixoX, EixoY	A = (0, 0)
2	Ponto B	Ponto sobre EixoX	B = (10, 0)
3	Ponto M	Ponto sobre EixoX	M = (4.68, 0)
4	Polígono pol1	Polígono(A, M, 4)	pol1 = 21.93
4	Segmento f	Segmento A, M	f = 4.68
4	Segmento g	Segmento M, C	g = 4.68
4	Ponto C	Polígono(A, M, 4)	C = (4.68, 4.68)
4	Ponto D	Polígono(A, M, 4)	D = (0, 4.68)
4	Segmento h	Segmento C, D	h = 4.68
4	Segmento i	Segmento D, A	i = 4.68
5	Polígono pol2	Polígono(M, B, 4)	pol2 = 28.27
5	Segmento j	Segmento M, B	j = 5.32
5	Segmento k	Segmento B, E	k = 5.32
5	Ponto E	Polígono(M, B, 4)	E = (10, 5.32)
5	Ponto F	Polígono(M, B, 4)	F = (4.68, 5.32)
5	Segmento l	Segmento E, F	l = 5.32
5	Segmento m	Segmento F, M	m = 5.32
6	Reta n	Reta A, C	n: $-4.68x + 4.68y = 0$
7	Reta p	Reta D, M	p: $4.68x + 4.68y = 21.93$
8	Ponto P	Interseção de n, p	P = (2.34, 2.34)
9	Reta q	Reta E, M	q: $5.32x - 5.32y = 24.9$
10	Reta r	Reta F, B	r: $5.32x + 5.32y = 53.17$
11	Ponto Q	Interseção de q, r	Q = (7.34, 2.66)
12	Círculo c	Círculo por A com centro P	c: $(x - 2.34)^2 + (y - 2.34)^2 = 10.97$
13	Círculo d	Círculo por B com centro Q	d: $(x - 7.34)^2 + (y - 2.66)^2 = 14.13$
14	Ponto N	Interseção de c, d	N = (4.37, 4.96)
15	Segmento s	Segmento A, N	s = 6.61
16	Segmento t	Segmento C, M	t = 4.68
17	Segmento a	Segmento B, N	a = 7.5
18	Segmento b	Segmento B, M	b = 5.32
19	Reta e	Reta M, N	e: $-4.96x - 0.31y = -23.23$
20	Círculo c1	Círculo por B com centro M	c1: $(x - 4.68)^2 + y^2 = 28.27$

Figure 11. The Olympics Teaching Situation Protocol conducted in GeoGebra

The protocol for this problem situation has aspects related to the perimeter and plane of the plane. The next topic exemplifies another parameterization in GeoGebra.

(c) Find the locus of the midpoint of the segment PQ when M varies between A and B:

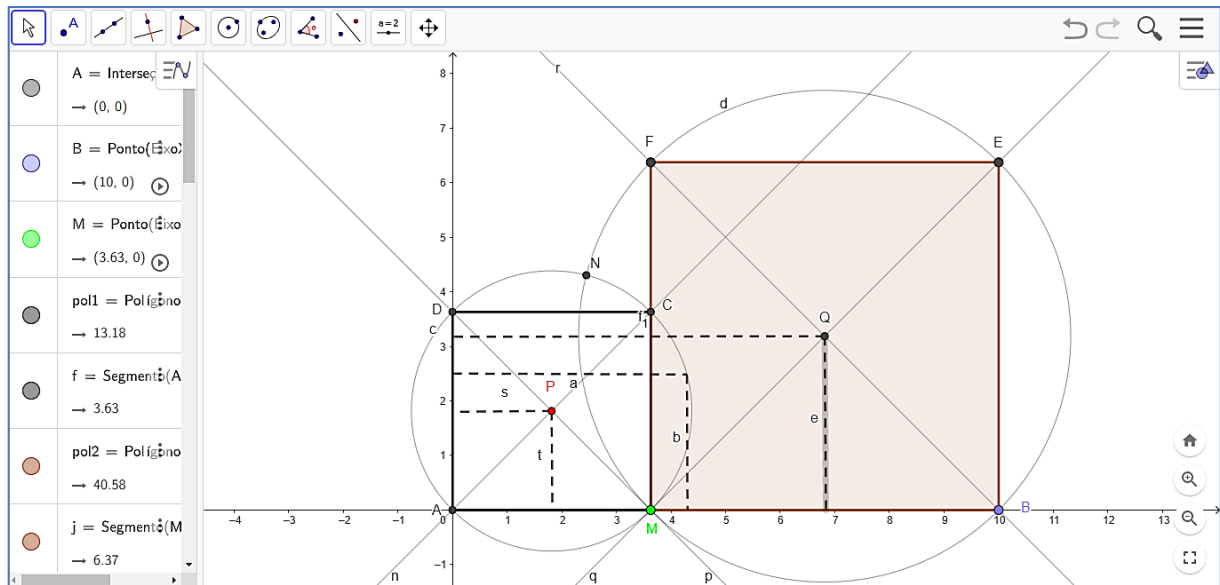


Figure 12. Points intercepted by the MBEF rectangle

Express the midpoint PQ as R. Obviously, the distance R from AB is the average of the distances P and Q from AB, half the length of AB, and is a constant. Therefore, the locus in question is a line segment.

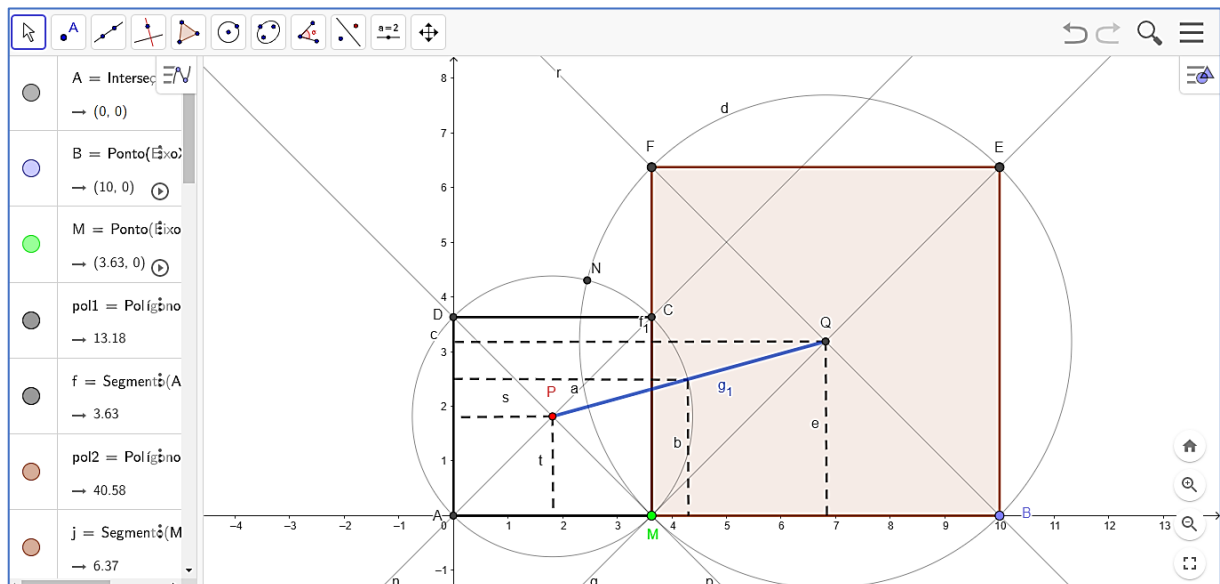


Figure 13. Line segments P and Q intersect other links from the x and y axes

The representation of structured commands in GeoGebra is shown in Figure 14, as shown below:

	Nome	Descrição	Valor			
1	Ponto A	Interseção de EixoX, EixoY	A = (0, 0)	6	Reta n	Reta A, C $n: -2.75x + 2.75y = 0$
2	Ponto B	Ponto sobre EixoX	B = (10, 0)	7	Reta p	Reta D, M $p: 2.75x + 2.75y = 7.56$
3	Ponto M	Ponto sobre EixoX	M = (2.75, 0)	8	Ponto P	Interseção de n, p $P = (1.37, 1.37)$
4	Polígono pol1	Polígono(A, M, 4)	pol1 = 7.56	9	Reta q	Reta E, M $q: 7.25x - 7.25y = 19.94$
4	Segmento f	Segmento A, M	f = 2.75	10	Reta r	Reta F, B $r: 7.25x + 7.25y = 72.5$
4	Segmento g	Segmento M, C	g = 2.75	11	Ponto Q	Interseção de q, r $Q = (6.37, 3.63)$
4	Ponto C	Polígono(A, M, 4)	C = (2.75, 2.75)	12	Círculo c	Círculo por A com centro P $c: (x - 1.37)^2 + (y - 1.37)^2 = 3.78$
4	Ponto D	Polígono(A, M, 4)	D = (0, 2.75)	13	Círculo d	Círculo por B com centro Q $d: (x - 6.37)^2 + (y - 3.63)^2 = 26.28$
4	Segmento h	Segmento C, D	h = 2.75	14	Reta s	Reta A, F $s: -7.25x + 2.75y = 0$
4	Segmento i	Segmento D, A	i = 2.75	15	Reta t	Reta B, C $t: -2.75x - 7.25y = -27.5$
5	Polígono pol2	Polígono(M, B, 4)	pol2 = 52.57	16	Ponto N	Interseção de c, s $N = (1.26, 3.32)$
5	Segmento j	Segmento M, B	j = 7.25	17	Ângulo α	Ângulo entre B, N, F $\alpha = 90^\circ$
5	Segmento k	Segmento B, E	k = 7.25	18	Ângulo β	Ângulo entre N, B, A $\beta = 20.77^\circ$
5	Ponto E	Polígono(M, B, 4)	E = (10, 7.25)			
5	Ponto F	Polígono(M, B, 4)	F = (2.75, 7.25)			
5	Segmento l	Segmento E, F	l = 7.25			
5	Segmento m	Segmento F, M	m = 7.25			

Figure 14. The construction command performed in GeoGebra at the end of the question

From the stages that have been carried out, students will learn from this ODS resolution regarding the use of property involved in other cases in the future, and we hope that teachers will guide students to provide meaningful learning in their interactions with peers. The GeoGebra software offers dynamics and visualization of the figure in question, making it an added attraction in the classroom.

To improve the teaching and learning process, it is included in the school curriculum in a well-articulated way, and can be modified depending on the didactic situation that the teacher will apply to the type of digital educational resources that can make the classroom better. dynamic, arousing interaction, interest, and commitment from students.

The problems presented in this IMO test demand from teachers and students an expressive level of mathematical knowledge and reasoning, making it inaccessible to everyone who wants to compete in international olympiads. Thus, there is a need to provide technological methods and tools that provide a way for teachers to incorporate them into their pedagogical planning. In connection with that matter Vieira & Pereira (2021) stressed the need for a deeper discussion of olympic issues arising from the Brazilian national Olympiad, as this brings content consistent with the school curriculum.

In this regard, GeoGebra's support in the perspective of structuring figures discussed in international issues, will provide teachers with an alternative for their daily planning or for use in preparing for Mathematics Olympiads and consequently, for use in the teaching of mathematics for classroom contexts and can be extended to the environment with the technological resources available in school institutions, enhancing the mediation and learning of students with international olympiad problems.

CONCLUSION

Through teaching theory supported by GeoGebra, students can understand the ODS constructive process. GeoGebra software assists in the elaboration of mathematical examples and problem solving situations and can be used in future activities, preparation for math olympiads and classroom learning by building learning through the dynamism provided by technological tools. In addition, the use in the classroom or in an environment with technological resources (computers) as an alternative means that is different from the traditional way of teaching, where the teacher has knowledge.

There are difficulties hindering the inclusion of these technological resources in the classroom as math teachers' methodologies change. Lack of special training for the development of activities leading to the use of mathematical competition problems in the school environment, absence of computers in school institutions (computer laboratories), lack of teacher planning time to enter Olympiads. mathematical problems and didactic transposition of these problems to the GeoGebra

software. Therefore, the didactic situation of the International Olympiad abstracted from IMO can be used by mathematics teachers for face-to-face teaching and for Olympic competitions. In addition, it is hoped that the material will stimulate pedagogical and methodological changes, thereby attracting more teachers to use IMO and other olympic issues.

AUTHOR CONTRIBUTION STATEMENT

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